Convolution model of altimeter waveforms

In order to investigate the effects of spatiotemporal variability in ice-sheet properties on radar altimeter measurements we apply a numerical deconvolution technique to CS2 Low Resolution Mode (LRM) waveforms, based upon the work of Arthern et al. (Arthern et al., 2001). The convolution model for waveforms is written as a convolution of three functions of time, t:

$$p(t) = p_t(t) * I(t) * s(t) \quad (1)$$

where $*$ denotes the convolution operation, $p(t)$ is the pulse received by the satellite, $p_t(t)$ is the transmitted pulse, $I(t)$ is the flat impulse response of the instrument to a given surface, and $s(t)$ is the distribution of scattering with depth. $s(t)$ contains all scattering contributions from surface topography which deviates from the given surface, and subsurface scattering resulting from penetration of the radar wave beyond the air-snow interface.

Convolution theorem states that the Fourier Transform of the convolution of multiple functions is equal to the product of their spectra in the frequency domain. Utilising this property equation 1 can be rewritten as:

$$p(t) = p_t(t) * I(t) * s(t) \leftrightarrow P_t(\omega)I(\omega)S(\omega) = P(\omega) \quad (2)$$

where '↔' denotes the Fourier transform operation. The purpose of the waveform deconvolution is to isolate $s(t)$, obtaining the distribution of scattering with depth. This is achieved simply by solving equation 2 for $s(t)$:

$$s(t) \leftrightarrow \frac{P(\omega)}{P_t(\omega)\overline{\overline{I(\omega)}}}\Pi(\omega) \quad (3)$$

In the present iteration of the model $P_t(\omega)\overline{I(\omega)}$ is estimated from the mean of LRM waveforms from a sea track obtained over the Mediterranean Sea. This empirical approach provides an approximation for the mean surface response of the ice-sheet (Brown, 1977). $\Pi(\omega)$ is a Gaussian low-pass filter with a standard deviation of 40 frequency bins, which is applied to eliminate high frequency artifacts resulting from the Gibbs phenomenon.

To obtain backscatter parameters $s(t)$ is fitted to the following function, described by equation 19 in Arthern et al. [2001], using a linear least squares regression:

$$y\left(t; \sigma_0^{surf} \sigma_0^{vol}, k_e, \gamma, \bar{t}\right) = \frac{\sigma_0^{surf}}{\gamma\sqrt{\pi}} e^{(t-\bar{t})^2/\gamma^2} + \frac{\sigma_0^{vol} c_{ice} k_e}{2} \cdot exp\left(\frac{\gamma^2 c_{ice}^2 k_e^2}{4} - c_{ice} k_e (t-\bar{t})\right) \cdot \left[1 + erf\left(\frac{(t-\bar{t})}{\gamma} - \frac{\gamma c_{ice} k_e}{2}\right)\right] \quad (4)$$

where the backscatter parameters of interest are surface backscatter, volume backscatter and extinction coefficient, denoted by σ_0^{surf}, σ_0^{vol} and k_e respectively. The units of σ_0^{surf} and σ_0^{vol} obtained from the deconvolution depend on the fact that the input CS2 and sea-track waveforms are normalised, therefore only relative differences are presented here.

References

Arthern, R.J. et al. 2001. Controls on ERS altimeter measurements over ice sheets: Footprint-scale topography, backscatter fluctuations, and the dependence of microwave penetration depth on satellite orientation. *Journal of Geophysical Research: Atmospheres*. **106**(D24),pp.33471–33484.

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